Act 1: Binary Session Types

With a deeply embedded binder representation.

Locally Nameless

with the Simply Typed Lambda-Calculus.

Terms $M, N ::= n \mid x \mid \lambda\,.\,M \mid M\,N$

Types $S, T ::= \texttt{base} \mid S \to T$

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Bound variables are represented by their De Bruijn index (i.e: a natural number).

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Bound variables are represented by their De Bruijn index (i.e: a natural number).

Free variables are represented by a name (i.e: an element of a nominal set).

Binders are anonymous (as with De Bruijn indices in general).

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with the Simply Typed Lambda-Calculus.

Terms $M, N ::= n \mid x \mid \lambda \,.\, M \mid M\ N$

Types $S, T ::= \texttt{base} \mid S \to T$

$$M^x \equiv \{0 \to x\}M$$ Open a term.

$$\backslash^x M \equiv \{0 \leftarrow x\}M$$ Close a term.

$$\texttt{lc}(M)$$ A locally closed term.

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Open a term.

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Close a term.

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A locally closed term.

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T}$$

$$\frac{\forall x \notin L \qquad \Gamma, x : S \vdash M^x : T}{\Gamma \vdash \lambda\,.\,M : S \to T}$$

$$\frac{\Gamma \vdash M : S \to T \qquad \Gamma \vdash N : S}{\Gamma \vdash M\;N : T}$$

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smolEMTST

A simple calculus with binary session types.

$$
\begin{array}{rrcl}
\text{Expressions} & e & ::= & \texttt{tt} \mid \texttt{ff} \mid () \mid x \\
\text{Sorts} & S & ::= & \mathit{bool} \mid \mathit{unit} \\
\text{Processes} & P, Q & ::= & k![e].P \mid k?().P \mid P \mid Q \\
& & & \mid \texttt{if}\ e\ \texttt{else}\ P\ \texttt{else}\ Q \\
& & & \mid \nu.P \mid !P \mid \texttt{inact} \\
\text{Types} & T & ::= & ?[S].T \mid ![S].T \mid \texttt{end} \mid \bot
\end{array}
$$

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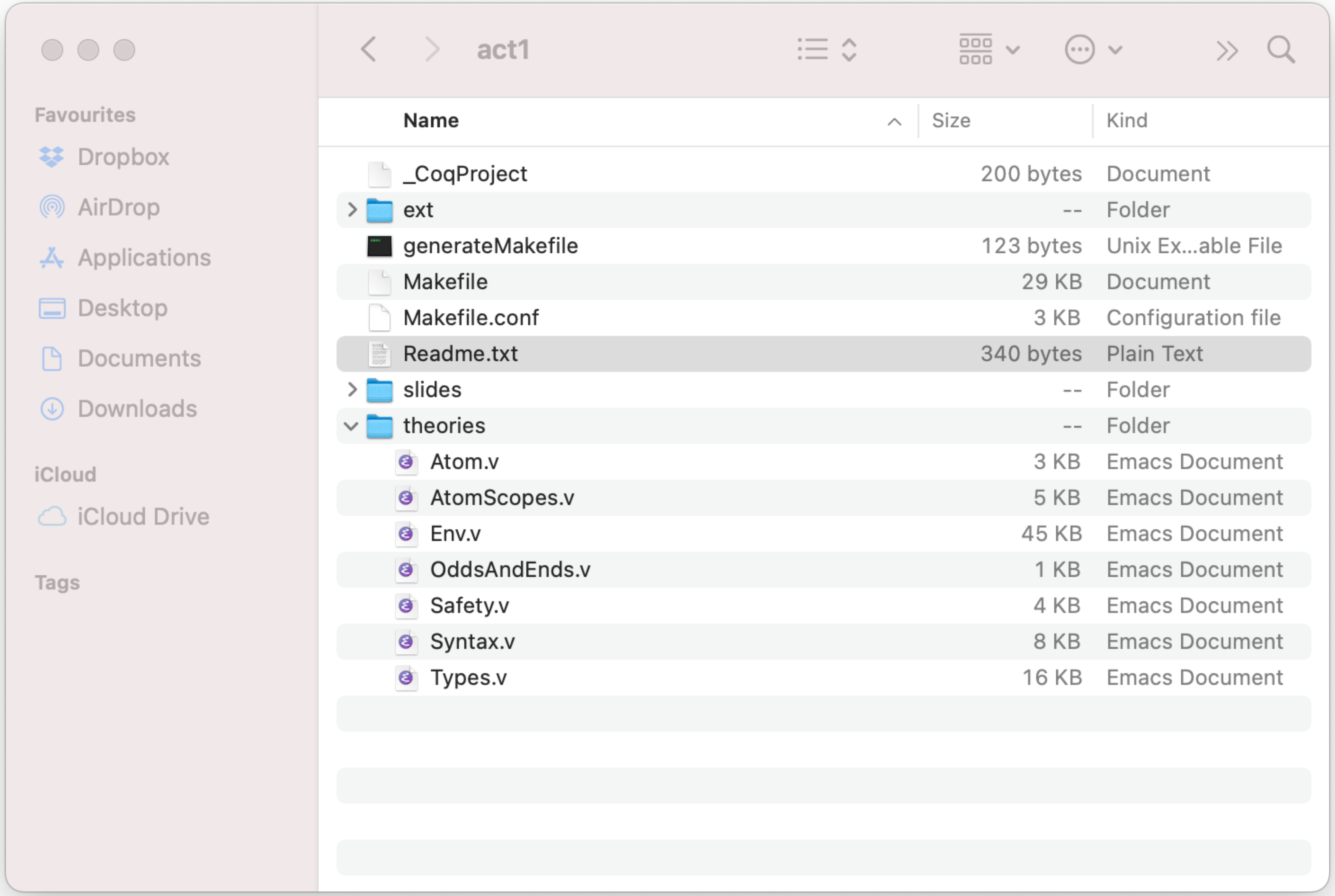
expression variables

channel variables

smolEMTST

At: http://github.com/emtst/gentleAdventure



smolEMTST

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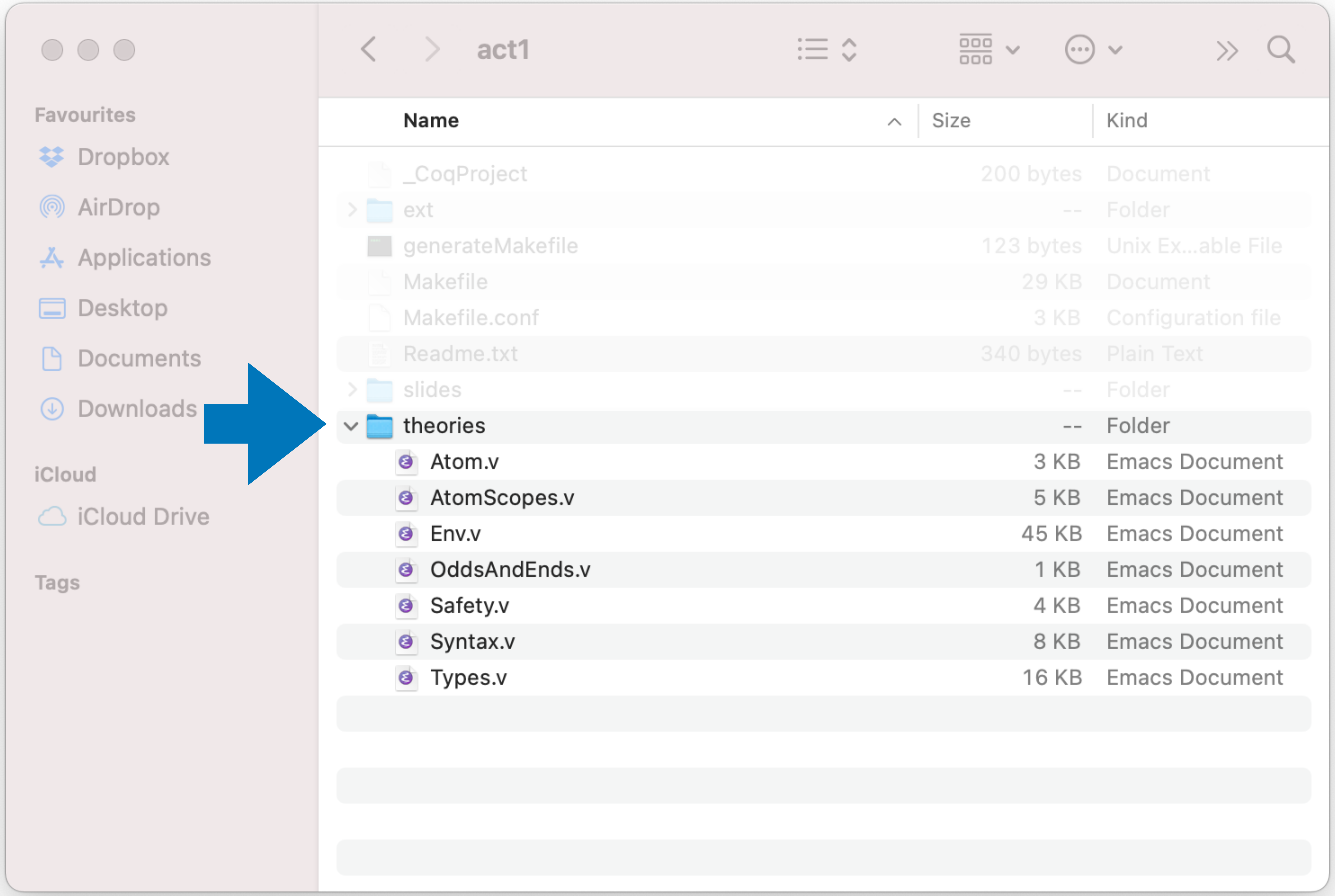

smolEMTST

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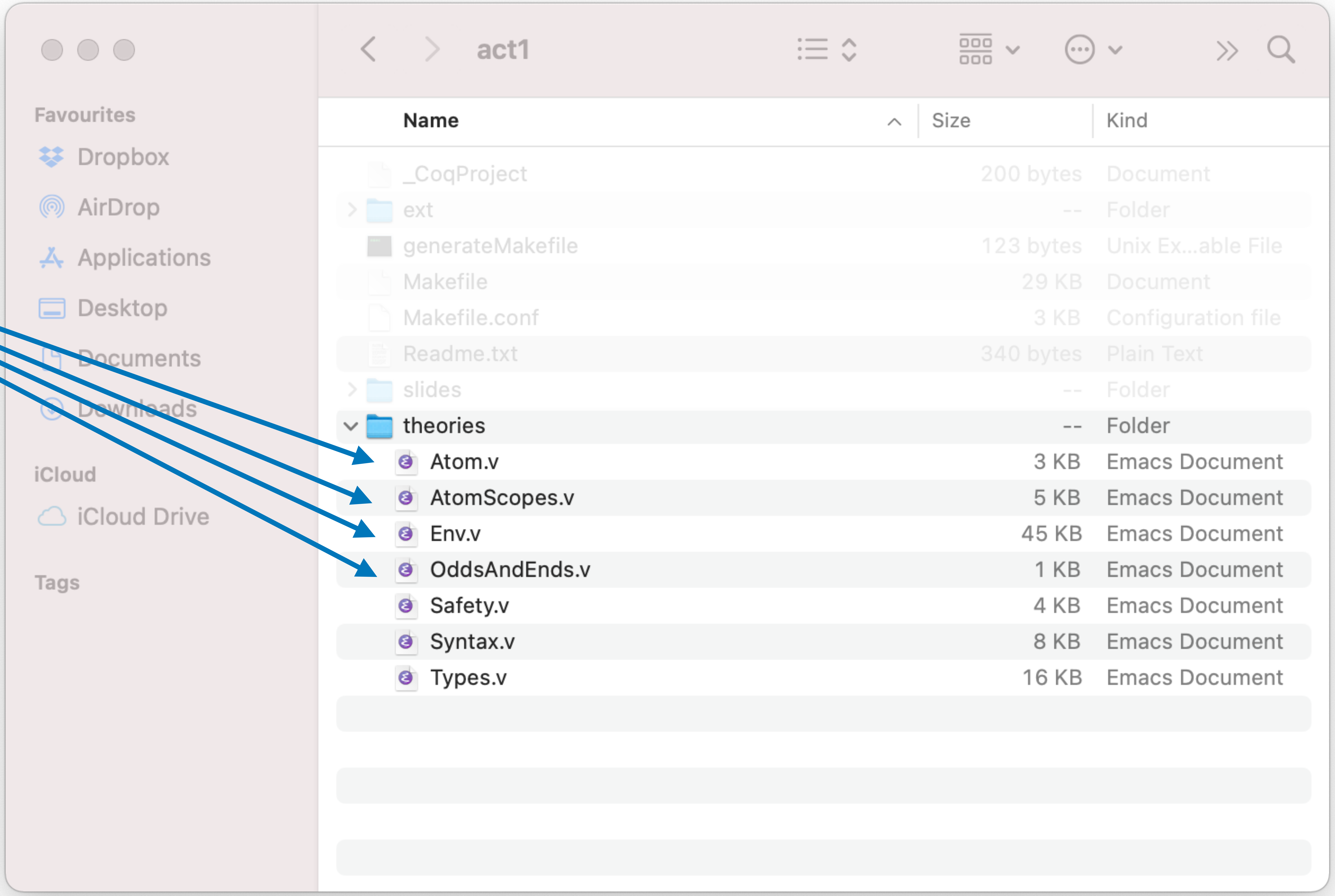
EMTST the tool:
- **multiple name scopes**
- **environment handling**



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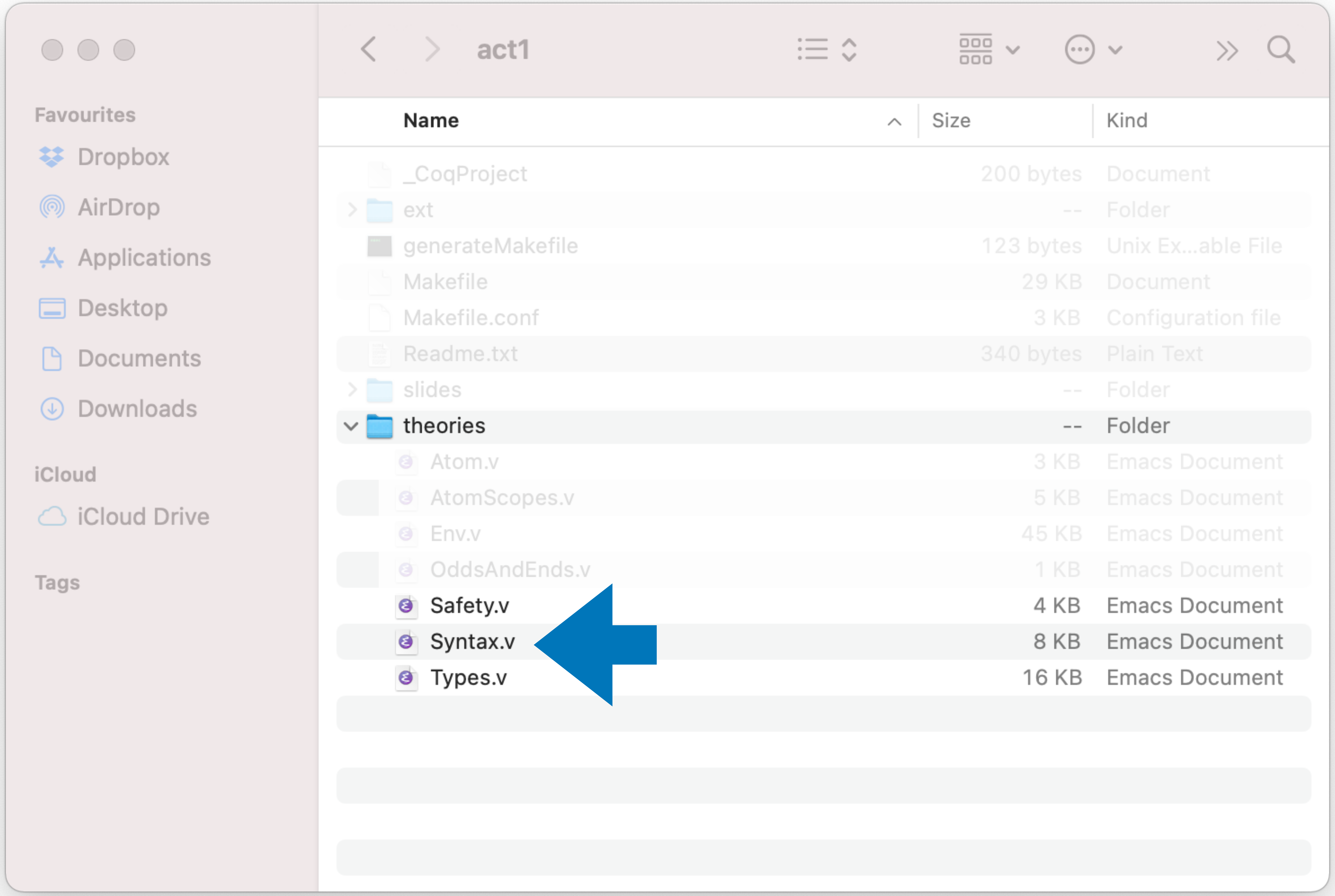

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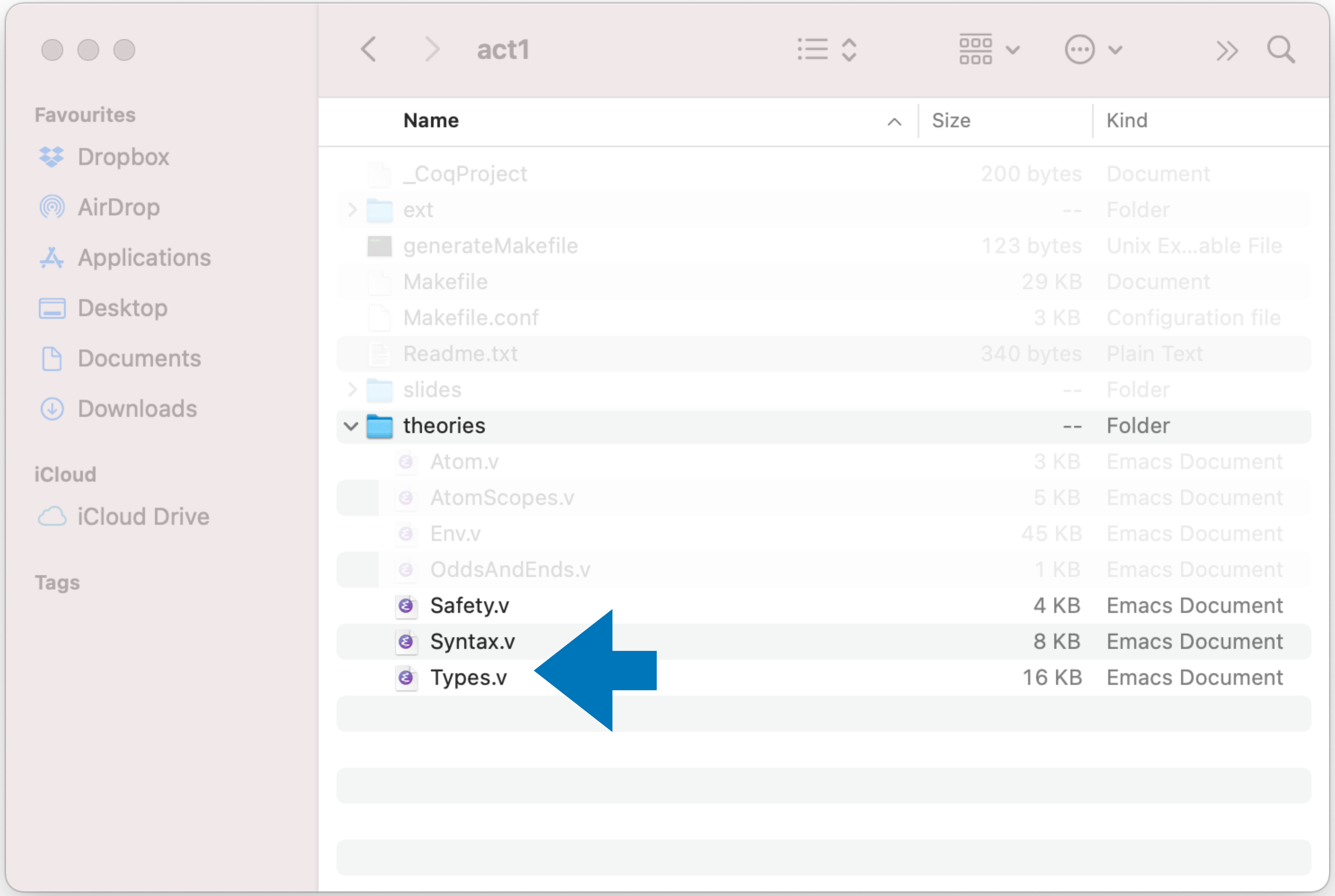

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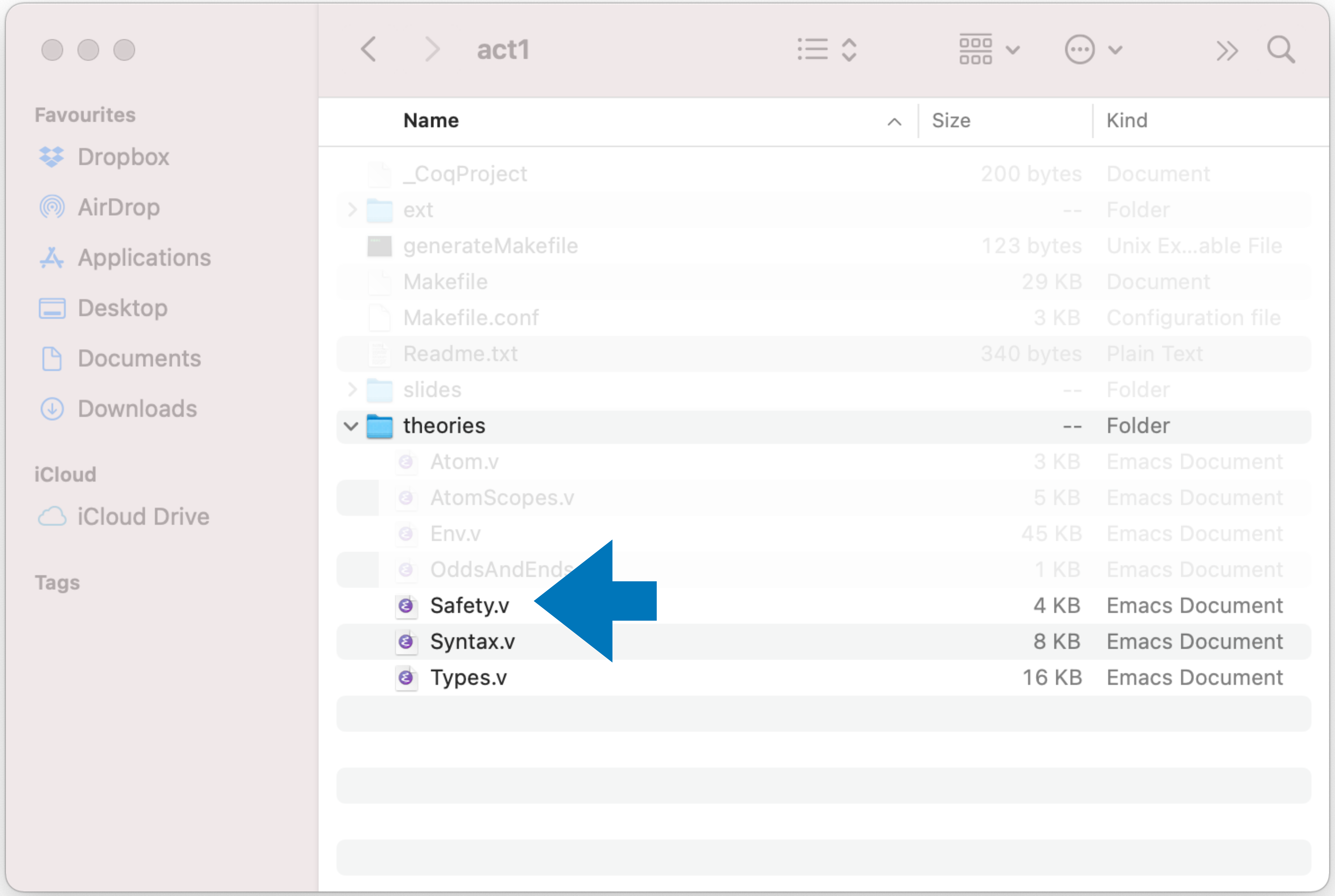

Representing the Syntax.

Locally closed brings the binding structure.

Expressions $e ::= \mathtt{tt} \mid \mathtt{ff} \mid () \mid x$

```
Inductive exp : Set :=
  | tt
  | ff
  | one
  | V of evar
.

Coercion V : evar >-> exp.
```

```
Inductive lc_exp : exp → Prop :=
  | lc_tt : lc_exp tt
  | lc_ff : lc_exp ff
  | lc_one : lc_exp one
  | lc_var a: lc_exp (V(EV.Free a))
.
```

```
(* Open a bound variable in an expression (original ope) *)
Definition open_exp (n : nat) (e' : exp) (e : exp) : exp :=
  match e with
  | V v ⇒ EV.open_var V n e' v
  | _ ⇒ e
  end.
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The Syntax of Processes

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$$\mid \texttt{if}\ e\ \texttt{else}\ P\ \texttt{else}\ Q$$
$$\mid \nu.P \mid !\,P \mid \texttt{inact}$$

```
Inductive proc : Set :=
| send : CH.var → exp → proc → proc
| receive : CH.var → proc → proc
| ife : exp → proc → proc → proc
| par : proc → proc → proc
| inact : proc
| nu : proc → proc
| bang : proc → proc
.
```

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.
```

```
Fixpoint open_e (n : nat) (u : exp) (P : proc) : proc :=
  match P with
```

```
Fixpoint open_k (n : nat) (ko : CH.var) (P : proc) : proc :=
  match P with
```

```
Inductive lc : proc → Prop :=
| lc_send : forall k e P,
    CH.lc k →
    lc_exp e →
    lc P →
    lc (send k e P)

| lc_receive : forall (L : seq EV.atom) k P,
    CH.lc k →
    (forall x, x \notin L → lc (open_e0 P (V (EV.Free x)))) →
    lc (receive k P)

| lc_nu : forall (L : seq CH.atom) P,
    (forall k, k \notin L → lc (open_k0 P (CH.Free k))) →
    lc (nu P)

| lc_bang P: lc P → lc (bang P)

(* ... *)
```

The Syntax of Processes

Remember, locally closed brings the binding structure.

$$\text{Processes } P, Q ::= k![e].P \mid k?().P \mid P \mid Q$$
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| bang : proc → proc
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```

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Fixpoint open_e (n : nat) (u : exp) (P : proc) : proc :=
  match P with
```

```
Fixpoint open_k (n : nat) (ko : CH.var) (P : proc) : proc :=
  match P with
  | send k e P ⇒ send (opk n ko k) e (open_k n ko P)
```

```
Inductive lc : proc → Prop :=
| lc_send : forall k e P,
    CH.lc k →
    lc_exp e →
    lc P →
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| lc_nu : forall (L : seq CH.atom) P,
    (forall k, k \notin L → lc (open_k0 P (CH.Free k))) →
    lc (nu P)

| lc_bang P: lc P → lc (bang P)

(* ... *)
```

Congruence and Reduction

Keeping an eye on locally nameless.

$$
\begin{array}{rl}
[\text{C-REFL}] & P \equiv P \\
[\text{C-INACT}] & P \mid \mathtt{inact} \equiv P \\
[\text{C-COMM}] & P \mid Q \equiv Q \mid P \\
[\text{C-ASSOC}] & (P \mid Q) \mid R \equiv P \mid (Q \mid R) \\
[\text{C-NU}] & \nu.(P \mid Q) \equiv \nu.P \mid Q \quad \text{if } \mathtt{lc}(Q) \\
[\text{C-NU'}] & \nu.\mathtt{inact} \equiv \mathtt{inact}
\end{array}
$$

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[C-REFL]	$P \equiv P$
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[C-COMM]	$P \mid Q \equiv Q \mid P$
[C-ASSOC]	$(P \mid Q) \mid R \equiv P \mid (Q \mid R)$
[C-NU]	$\nu.(P \mid Q) \equiv \nu.P \mid Q$ if $\texttt{lc}(Q)$
[C-NU']	$\nu.\texttt{inact} \equiv \texttt{inact}$

[R-COM]	$k![e].P \mid k?().Q \to P \mid Q^e$
[R-CONG]	$P \equiv P'$ and $P' \to Q'$ and $Q' \equiv Q \Rightarrow P \to Q$
[R-SCOP]	$P \to Q \Rightarrow \nu.P \to \nu.Q$
[R-PAR]	$P \to P' \Rightarrow P \mid Q \to P' \mid Q$
[R-IF-TT]	$\texttt{if tt else } P \texttt{ else } Q \to P$
[R-IF-FF]	$\texttt{if ff else } P \texttt{ else } Q \to Q$

Congruence and Reduction

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$$[\textsc{C-Refl}] \quad P \equiv P$$

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$$[\textsc{R-Com}] \quad k![e].P \mid k?().Q \to P \mid Q^{e}$$

$$[\textsc{R-Cong}] \quad P \equiv P' \text{and } P' \to Q' \text{and } Q' \equiv Q \Rightarrow P \to Q$$

$$[\textsc{R-Scop}] \quad P \to Q \Rightarrow \nu.P \to \nu.Q$$

$$[\textsc{R-Par}] \quad P \to P' \Rightarrow P \mid Q \to P' \mid Q$$

$$[\textsc{R-If-TT}] \quad \texttt{if}\ \texttt{tt}\ \texttt{else}\ P\ \texttt{else}\ Q \to P$$

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$$[\textsc{R-Scop}] \quad \forall k \notin L \quad P^k \to Q^k \Rightarrow \nu.P \to \nu.Q$$

$$[\textsc{R-Par}] \quad P \to P' \Rightarrow P \mid Q \to P' \mid Q$$

$$[\textsc{R-If-TT}] \quad \texttt{if tt else } P \texttt{ else } Q \to P$$

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[C-NU']	$\nu.\texttt{inact} \equiv \texttt{inact}$

[R-COM]	$k![e].P \mid k?().Q \to P \mid Q^e$
[R-CONG]	$P \equiv P' \text{and } P' \to Q' \text{and } Q' \equiv Q \Rightarrow P \to Q$
[R-SCO	$\forall k \notin L \quad P^k \to Q^k \Rightarrow \nu.P \to \nu.Q$
[R-PAR]	$P \to P' \Rightarrow P \mid Q \to P' \mid Q$
[R-IF-TT]	$\texttt{if } \texttt{tt} \texttt{ else } P \texttt{ else } Q \to P$
[R-IF-FF]	$\texttt{if } \texttt{ff} \texttt{ else } P \texttt{ else } Q \to Q$

Representing Judgments

It’s really nothing new.

```
Reserved Notation "P ⟶ Q" (at level 70).
Inductive red : proc → proc → Prop :=
| r_com (k : CH.atom) e P Q:
    lc P →
    (par (send k e P) (receive k Q)) ⟶ (par P ({ope 0 ↝ e} Q))

| r_cong P P' Q Q' :
    lc P → lc Q →
    P ≡ P' →
    P' ⟶ Q' →
    Q' ≡ Q →
    P ⟶ Q

| r_scop P P':
    (forall (L : seq CH.atom) k,
        k \notin L → (open_k0 P (CH.Free k)) ⟶ (open_k0 P' (CH.Free k))) →
    nu P ⟶ nu P'

| r_par P P' Q:
    lc Q →
    P ⟶ P' →
    par P Q ⟶ par P' Q

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where "P ⟶ Q" := (red P Q).
```

Representing Judgments

It's really nothing new.

```
Reserved Notation "P ⟶ Q" (at level 70).
Inductive red : proc → proc → Prop :=
| r_com (k : CH.atom) e P Q:
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[R-COM]	$k![e].P \mid k?().Q \to P \mid Q^e$
[R-CONG]	$P \equiv P' \text{and } P' \to Q' \text{and } Q' \equiv Q \Rightarrow P \to Q$
[R-SCOP]	$\forall k \notin L \quad P^k \to Q^k \Rightarrow \nu.P \to \nu.Q$
[R-PAR]	$P \to P' \Rightarrow P \mid Q \to P' \mid Q$
[R-IF-TT]	$\texttt{if tt else } P \texttt{ else } Q \to P$
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Typing smolEMTST

Locally nameless became easy by now.

[T-SEND]

$$\frac{\Gamma \vdash_e e : S \qquad \Gamma \vdash P : \Delta \cdot k : T}{\Gamma \vdash k![e].P : \Delta \cdot k : ![S].T}$$

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$$\frac{\Gamma \vdash P : \Delta \qquad \Gamma \vdash P : \Delta' \qquad \Delta \asymp \Delta'}{\Gamma \vdash P \mid Q : \Delta \circ \Delta'}$$

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$$\frac{\Gamma \vdash_e e : bool \qquad \Gamma \vdash P : \Delta \qquad \Gamma \vdash Q : \Delta}{\Gamma \vdash \texttt{if}\ e\ \texttt{else}\ P\ \texttt{else}\ Q : \Delta}$$

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Substitution Lemmata

Because one lemma is not enough.

```
Lemma SubstitutionLemmaExp G x S S' e e':
  binds x S' G →
  oft_exp G e' S' →
  oft_exp G e S → oft_exp G (s[ x ↝ e']e e) S.
Proof.
  move⇒Hbind Hde' Hde.
  move:Hde'.
  elim Hde ; try constructor ; try assumption.
  intros.
  case: (EV.eq_reflect x x0).
  move⇒Sub.
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  rewrite eq_refl.
  have Heq : S' = S0 by apply: UniquenessBind ; [apply: Hbind | apply: H].
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Qed.
```

```
Theorem ExpressionReplacement G P x E S D:
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Proof.
Admitted.
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Substitutes expressions in processes

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Proof.
Admitted.
```

```
Lemma ChannelReplacement G P c c' D :
  oft G P D →
  def (subst_env c c' D) →
  oft G (s[ c ↝ chan_of_entry c' ]p P) (subst_env c c' D).
Proof.
  case: (boolP (c' == c)) ⇒[/eqP→/⟸|c_neq_c']; (* ... *)
```

Subject Reduction

These proofs will motivate some other lemmas (e.g: weakening).

```
Theorem CongruencePreservesOft G P Q D :
  P ≡ Q → oft G P D → oft G Q D.
Proof.
  elim⇒{P}{Q}//. (* ... *)
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Theorem SubjectReductionStep G P Q D:
  oft G P D → P ⟶ Q → exists D', D ⇝ D' ∧ oft G Q D'.
Proof.
  move⇒ Op PQ.  (* ... *)
```

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Theorem SubjectReduction G P Q D:
  oft G P D → P ⟶* Q → exists D', oft G Q D'.
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  move ⇒ Hoft PQ; elim: PQ D Hoft ⇒ {P} {Q} P.
  + by move⇒ D Hoft; exists D.
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    move: (SubjectReductionStep Hoft Step) ⇒ []D' []bD' Hoft'.
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Conclusion of the First Act.

No intermediate and no tiny ice cream like at the theatre.

- Deep embedding (LN) binders allows us to fully control the calculus.
- LN demands tribute for that control (in the shape of theorems).
- EMTST (the tool) helps with nominal sets and environments.
- **In the next act we explore what do we get if we give up control (using shallow embeddings).**